Solid State Physics (PCS 224): Notes

Adam Szava

Fall 2021

Introduction

This is my compilation of notes from Solid State Physics (PCS 224) from Ryerson University. All information comes from my professor's lectures, the textbook *Physics for Scientists and Engineers*, and online resources.

Lesson 1: Electrostatics Review

Charged objects cause an electric field around them. The electric field is a vector field pointing away from the positive source charges, or towards negative source charges. The magnitude of the electric field is:

$$\left|\vec{E}\right| = \frac{k_e|q|}{r^2}$$

... where q is the charge of the source charge, r is the radial distance from the source charge, and k_e is the proportionality constant of:

$$\boxed{k_e = 9 \times 10^9 \frac{N \cdot m^2}{C^2}}$$

A particle in the field with charge q will feel an electric force of:

$$\vec{F}_e = q\vec{E}$$

If there are multiple source charges, then the electric field at any point is the sum of the two electric field vectors from each source charge.

Close to a large plane of charge, the electric field is perpendicular to the plane with magnitude:

$$\left|\vec{E}\right| = \frac{Q}{2A\varepsilon_0}$$

... where Q is the charge of the plane, A is the area of the plane, and ε_0 is the permittivity of free space:

$$\boxed{\varepsilon_0 = \frac{1}{4\pi k_e} = 8.85 \times 10^{-12} \frac{C^2}{N \cdot m^2}}$$

The electric force is a conservative force, and so total energy is conserved for a particle moving under the influence of the electric force. For a particle moving in a uniform electric field:

$$|\Delta U_E| = |q|\left|\vec{E}\right||\Delta r|$$

... where Δr is the distance travelled in the direction of the electric field.

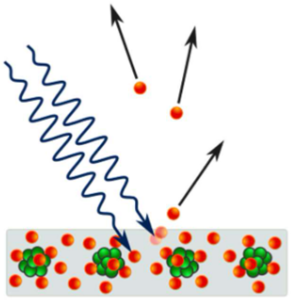
The *electron volt* (eV) is a measure of energy in J, defined as:

$$1eV = 1.602 \times 10^{-19} J$$

This is the amount of energy gaind by a single electron accelerating from rest through a potential difference of one volt.

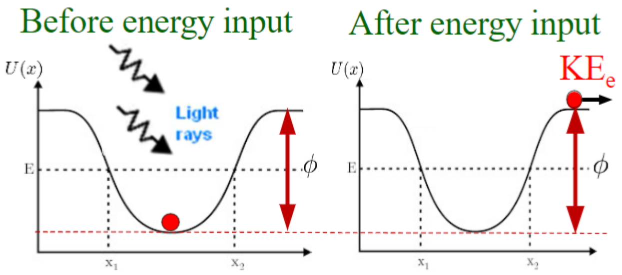
Lesson 2: Photoelectric Effect

The photoelectric effect is the effect of light shining on metal, causing electrons to get knocked off.



An electron in a low energy state within a metal can be excited to a higher energy state by light, which if that excitation energy is high enough, can cause the electron to leave the metal even with some kinetic energy.

The amount of energy needed to release the electron is a property of the metal called the *work function* (ϕ), illustrated as:



By this logic we can see that the energy of the light coming to hit the metal (E_{light}) is:

$$E_{light} = KE_e + \phi$$

Light is a wave, with frequency, wavelength, and speed, related like any wave as:

$$v = \lambda f$$

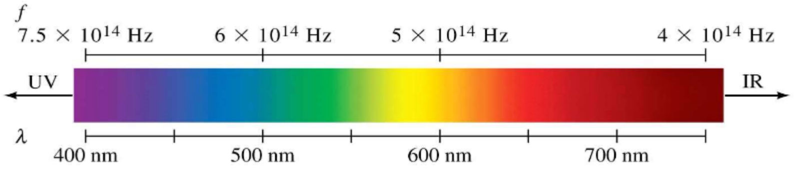
... except the speed of light is a constant:

$$v = c = 3 \times 10^8 \frac{m}{s}$$

... so the following equation is always true for light waves:

$$\boxed{f\lambda = c}$$

Different frequencies correspond to different colors of light:



Experimentation of the Photoelectric Effect

The photoelectric effect can be measured by an experiment which contains a circuit with an ammeter, a battery which supplies the electrons in the circuit with voltage, and a gap in the circuit which electrons will pass through when exposed to light. One side of the gap is called the emitter, and the other is called the collector.

Before light is exposed onto the photo sensitive cathode of the gap, there is no current since there is just a gap in the circuit.

If you expose the gap to light, then some electrons will begin to jump across the gap and the ammeter will read a non-zero current. This only happens if the frequency of light is above some minimum threshold called the *cutoff frequency.*

If you flip the battery so that the collector is now the negative plate, the value of the reversed voltage V_{stop} that prevents the electrons from reaching the collector is called the stopping potential. This happens because the electrons are repelled by the negative plate. This is the voltage difference across the emitter and collector which makes no electrons reach the collector.

The following equation is true regarding the stopping potential and then kinetic energy of the electron leaving the collector:

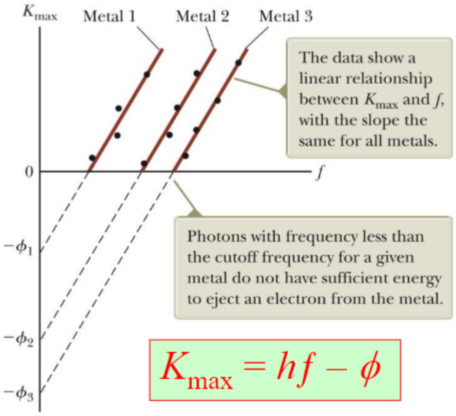
$$e\Delta V_{stop} = KE_e$$

Results of Experiment

1. No electrons are ejected below the cutoff frequency.

$$\boxed{\phi = hf_c}$$

2. Kinetic energy is independent of light intensity, but increases linearly with increasing light frequency.



The data show a linear relationship between K_{max} and f, with the slope the same for all metals.

Photons with frequency less than the cutoff frequency for a given metal do not have sufficient energy to eject an electron from the metal.

$$K_{max} = hf - \phi$$

3. Electrons start to emit almost instantly regardless of light intensity.

The Particle Model of Light

This lead to the particle model of light where:

1. All electromagnetic radiation can be considered a stream of *quanta*, a beam of bundles of energy called *photons*, each with energy:

$$hf$$

... where:

$$h \text{ (Planck's Constant)} = 6.626 \times 10^{-34} J \cdot s$$

... and f is the frequency of the photon.

2. A photon hitting an incident electron, giving it all it's energy. If that energy is enough to release the electron from its material, then it does potentially with some left over kinetic energy (K_{max}).

$$\boxed{E_{photon} = hf = \frac{hc}{\lambda} = K_{max} + \phi}$$

3. When light intensity increases, so does the number of photons hitting the emitter, which causes the current to increase.

Lesson 3: Semiconductors

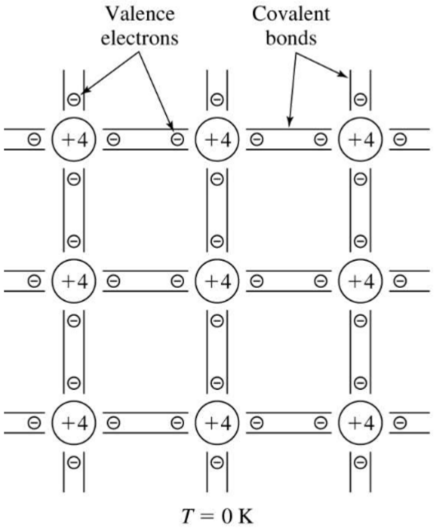
In an insulator, all electrons are bound to atoms and cannot move freely through materials. In a conductor, some electrons are free and can move relatively freely through the material.

In a *semiconductor* the electric property is somewhere between an insulator and a conductor. Examples of semiconductors include:

$$Si, Ge, GeAs$$

Naturally occurring semiconductor are called *intrinsic* semiconductors. Adding impurities makes them *extrinsic* semiconductors.

Focusing on silicon semiconductors, they form four covalent bonds which each take $1.12eV$ to break. At $0K$, there is no additional energy, and so no free electrons as current carriers, meaning it is an insulator.

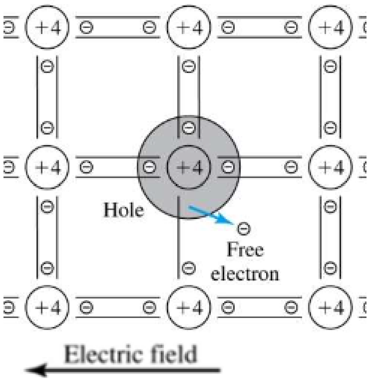


Temperature is defined as the measure of the random thermal motion of a large number of particles. *Room temperature* is defined as $300K$, or 27° Celsius by the following formula:

$$T_K = T_C + 273$$

Current Carrying Semiconductors

At $0K$ the silicone semiconductor is an insulator, but at non-zero temperatures, the electrons begin to break free from the bonds and knock other electrons out of place. They move *against* the electric field applied to the semiconductor. Every electron leaves behind a *hole* which is a positively charged region of space.



Holes

Holes move in the same direction as the electric field, and so are *equivalent* to positive charges.

We model the holes as carriers of electric charge, while they really represent the collective motion of many electrons.

Carrier Concentration

1. n_i is the number of free electrons per unit volume in an intrinsic semiconductor.
2. p_i is the number of holes per unit volume in an intrinsic semiconductor.
3. n_0 is the number of free electrons per unit volume in any semiconductor in thermal equilibrium.
4. p_0 is the number of free electrons per unit volume in any semiconductor in thermal equilibrium.

They are also called *concentration* or *number density* and are all measured in $1/cm^3$ For an intrinsic semiconductor:

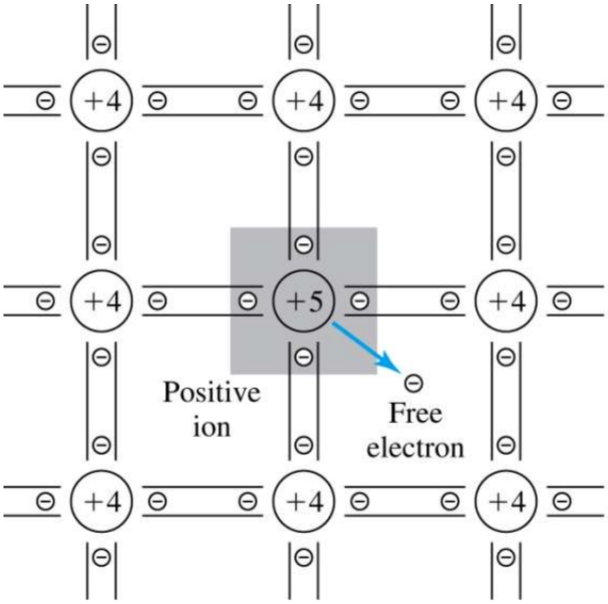
$$\boxed{n_i = p_i} = n_0 = p_0$$

Extrinsic Doping

The addition of impurities in a semiconductor to alter its electrical properties is called doping. These impurities take the place of some atoms in the silicone structure. There are two types of doping:

n-Type

n-Type doping is where some silicone atoms are replaced with a *Group 5* element (like Phosphorus), which contain five valence electrons, one more than silicone.

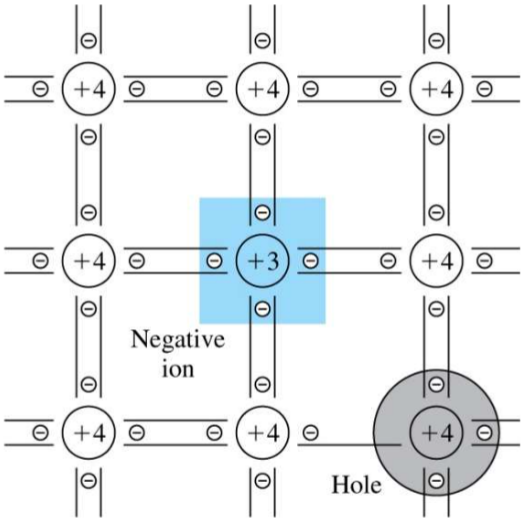


N-type doping

This causes one electron to be added to the structure giving this doping the name *negative-type* or just *n-type*. The extra electrons are called *donor* electrons and their concentration per unit volume is denoted N_d.

p-Type

p-Type doping is where some silicone atoms are replaced with a *Group 4* element (like Boron), which contain four valence electrons, one less than silicone.



P-type doping

This causes one hole to be added to the structure giving this doping the name *positive-type* or just *p-type*. Each impurity atom is called an acceptor, which provide extra holes. The concentration of acceptors per unit volume is denoted N_a.

Comparing Doped Materials

Any semiconductor in which:

$$n_0 > p_0$$

is *n-type*. Here there are more electrons than holes, and electrons are the majority charge carrier.

Any semiconductor in which:

$$n_0 < p_0$$

is *p-type*. Here there are more holes than electrons, and holes are the majority charge carrier.

In thermal equilibrium, the rate of generation of electron-hole pairs ($R_g(T)$) is equal to the rate of recombination ($R_r(T)$) (where an electron meets a hole and they annihilate).

Rate of generation depends only on material type and temperature.

The rate of recombination depends on n_0 and p_0, and so for an extrinsic semiconductor:

$$R_g(T) = \alpha n_0 p_0$$

... or for an intrinsic semiconductor:

$$R_g(T) = \alpha n_i^2$$

... which tells us that for a given material and temperature:

$$\boxed{n_0 p_0 = n_i^2}$$

Lesson 4: Current

Electric current I is defined as the rate of flow of charge through a region of space:

$$I = \frac{dQ}{dt} = \frac{\Delta Q}{\Delta t}$$

It is conventional to assign the current to be the direction of flow of positive charges. The current caused by positive charges flowing to one direction is equivalent to the flow of negative charges moving in the opposite direction.

In a typical conductor experiencing no influence of an electric field, charge carriers have frequent collisions and move randomly, although undergo no net displacement. In the presence of an electric field however, the charge carriers do experience a net displacement through the collisions and move with a small velocity called the *drift velocity* (v_d). Typically this value is around $10^{-4} m/s$.

Current and Charge Density

Charges move through a cross sectional area A in a wire of length Δx, where the number of charge carriers per unit volume is denoted n. Then:

$$nA\Delta x$$

... denotes the total number of charges in the section of wire. Therefore:

$$\Delta Q = q(nA\Delta x)$$

... denotes the total charge in the section of wire assuming all the particles are of the same charge. We know the particles have some drift charge:

$$v_d = \frac{\Delta x}{\Delta t} \implies \Delta x = v_d \Delta t$$

We can combine this with our formula for charge to get:

$$\Delta Q = qnAv_d\Delta t \implies \frac{\Delta Q}{\Delta t} = I = nqAv_d$$

Current density (J) is defined as:

$$J = \frac{I}{A} = \frac{nqAv_d}{A} = nqv_d$$

Conductivity

Conductivity (σ) is defined in:

$$J = \sigma E$$

Meaning the higher the conductivity, the better the flow of charge for a given electric field.

How fast charges are moving in a wire depend on the electric field, the proportionality constant is μ_p for holes and μ_n for electrons, as in:

$$\vec{v}_{dp} = \mu_p \vec{E}$$

$$\vec{v}_{dn} = -\mu_n \vec{E}$$

For n negative charges and p holes:

$$\vec{J}_{tot} = \vec{J}_n + \vec{J}_p = ne\mu_n \vec{E} + pe\mu_p \vec{E} = e(n\mu_n + p\mu_p)\vec{E}$$

... which gives us:

$$\vec{J}_{tot} = e(n\mu_n + p\mu_p)\vec{E}$$

Meaning:

$$\sigma = e(n\mu_n + p\mu_p)$$

Resistivity

Resistivity (ρ) is defined as:

$$\rho = \frac{1}{\sigma}$$

Given some conductor of length L and cross-sectional area A.

Take the definition of electric field:

$$\left|\vec{E}\right| = \frac{|\Delta V|}{L}$$

$$|\Delta V| = \left|\vec{E}\right| L$$

... taking $\vec{J} = \rho \vec{E}$

$$|\Delta V| = \left(\frac{L}{\sigma A}\right) I$$

This is *Ohm's Law*, if we define *resistance* (R) as:

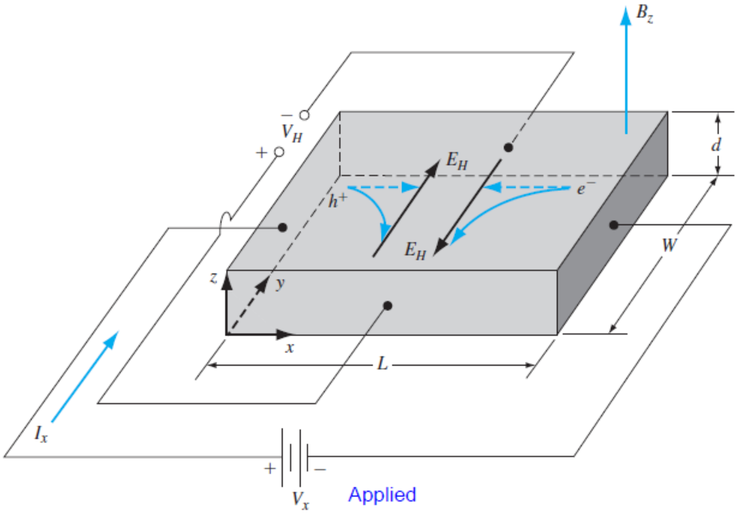
$$R \equiv \frac{\Delta V}{I} = \frac{\rho L}{A}$$

Lesson 5: The Hall Effect

The Hall effect is caused by a semiconductor in a magnetic field which has some voltage across its length causing an electric field within it. Recall the equation:

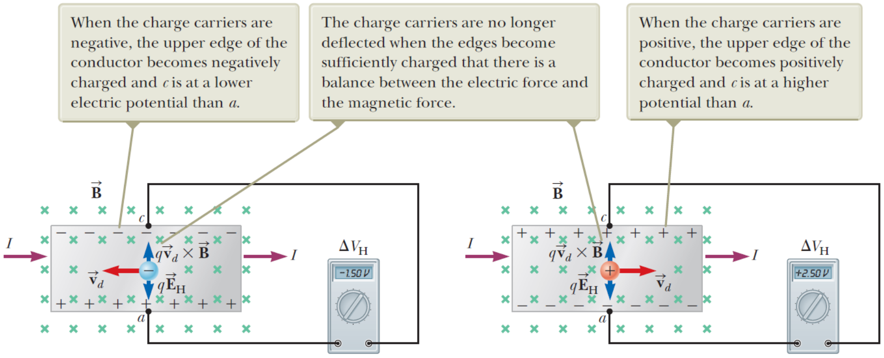
$$\vec{F_e} = q\vec{v} \times \vec{B}$$

In the following image you can see that the semiconductor has a voltage applied to it V_x.



This causes both the positive holes and the negative electrons to both build up on the same side of the semiconductor. You can then measure a potential difference across the semidonductor called the *Hall Voltage* V_H. If that voltage is positive, then the primary charge carrier was holes, otherwise the primary charge carrier was electrons. This method is used to determine if a semiconductor is p-type or n-type.

The particles continue to build up until the force from the electric field caused by the hall voltage E_H equalizes the magnetic force.



When the charge carriers are negative, the upper edge of the conductor becomes negatively charged and *c* is at a lower electric potential than *a*.

The charge carriers are no longer deflected when the edges become sufficiently charged that there is a balance between the electric force and the magnetic force.

When the charge carriers are positive, the upper edge of the conductor becomes positively charged and *c* is at a higher potential than *a*.

We can analyse the Hall effect to find information about our semiconductor.

Since the Hall field force equalizes the magnetic force:

$$qv_dB = qE_H \implies \boxed{E_H = v_dB}$$

If w is the width of the semiconductor parallel to the Hall field:

$$\Delta V_H = E_H w \implies \boxed{\Delta V_H = v_dBw}$$

Using this last equation you can find the drift speed if you can measure the Hall voltage, magnetic field, and width of semiconductor.

Analysing the current and drift speed relationship gets you:

$$\boxed{B = \frac{nqd}{I}\Delta V_H}$$

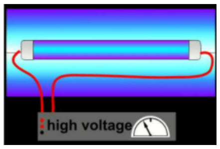
Where d is the thickness of the semiconductor in the direction of the magnetic field. With this equation you can measure the majority charge carrier concentration.

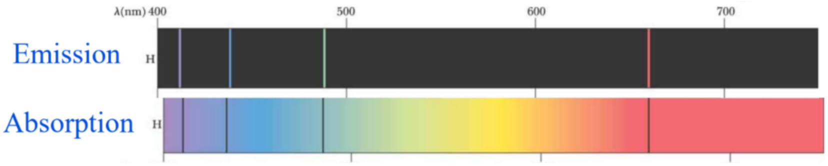
With the following equation you can find the mobility of the majority charge carriers:

$$\boxed{\mu = \frac{IL}{nWd|q||V_x|}}$$

Lesson 6: Bohr's Model

When a low pressure gas is subjected to an electric discharge, discrete wavelengths of light are emitted which are unique to every elemental gas. This analysis is called *emission spectroscopy*.





The Balmer series was empirically discovered and correctly calculates the particular wavelengths of light emitted by hydrogen:

$$\frac{1}{\lambda} = R_H \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right)$$

Where:

- $$n_1 = 1, 2, 3, \ldots$$
- $$n_2 = n_1 + 1, n_2 + 2, n_3 + 3, \ldots$$

Particular n_1 values relate to different sections of the radioactive spectrum.

Bohr's model of the atom states that electrons can orbit the nucleus so long as it's angular momentum is some integral multiple of $\hbar = \frac{h}{2\pi}$, as in:

$$m_2 v r_n = n \frac{h}{2\pi}$$

From this equation we can derive an expression for the n^{th} radius (r_n):

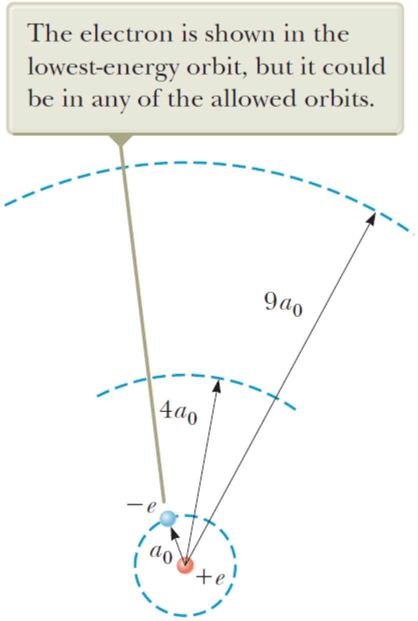
$$\boxed{r_n = a_0 n^2}$$

... where $a_0 = 0.0529nm$.

You can also derive an equation for the total energy of an atom (electron proton pair):

$$E_n = \frac{-13.6eV}{n^2}$$

As you can see the energy of an atom is not continuous can can only take on values dependent on integer n. We say that the energy of an atom is *quantized*.



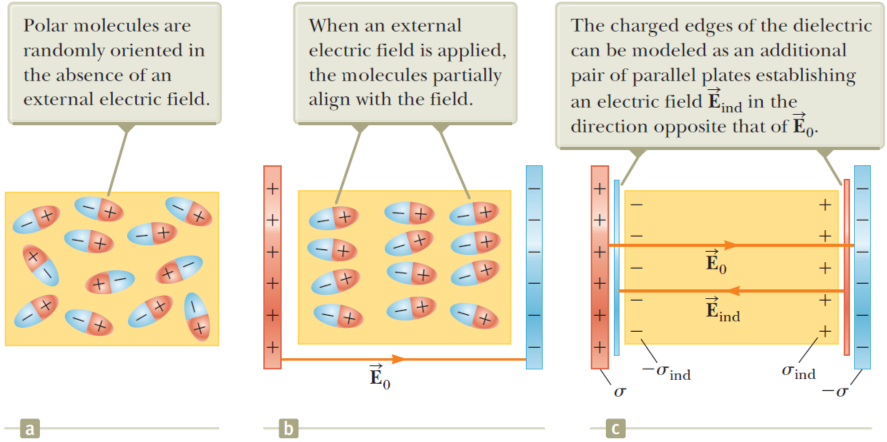
The lowest possible energy state is called the ground state. Electrons can move between states. They can increase their energy states by colliding with a photon, absorbing its energy only if it is exactly the amount required to jump up one or more levels. As the electron gets further from the nucleus it becomes more unstable and will release its energy by releasing a photon with energy: hf, meaning:

$$hf = E_i - E_f$$

Which can be used to find the frequency of the light emitted.

Lesson 7: Doping

Inside a material, molecules can become polarized meaning they align with the electric field due to their internal separation of charge. This causes them to line up and many of the internal charges cancel leaving the positive and negative end of the molecules on opposite ends creating an induced electric field:



The factor by which the field decreases is called the *dielectric constant* κ.

$$\left|\vec{E}\right|_{material} = \frac{\left|\vec{E}\right|_{vacuum}}{\kappa}$$

This causes electric fields to be decreased within materials. We can account for this by adjusting ε_0 and using $\kappa\varepsilon_0$. κ can be determined from a table of values.

Summary for Donors

1. $$n_0 > p_0$$
2. Binding energy of Silicon electrons $\approx 1.12eV$.
3. Binding energy of donor electrons $\approx 0.01 - 0.1eV$.

Since the binding energy of donor atoms is so low, they can be freed very easily at room temperature and so we assume all donors are ionized (meaning freed from the Silicon).

Donor concentration is denoted N_d and so we assume that all of those donor electrons are ionized so:

$$n_0 \approx N_d$$

Summary for Acceptors

1. $$n_0 < p_0$$

Hole are very easily filled at room temperature and so we assume all acceptor states have been filled. Acceptor state concentration is denoted N_a and because of our assumption we can also assume:

$$p_0 \approx N_A$$

Compensated Semiconductors

These types of semiconductors are both n-type and p-type. The type of material is determined by the doping with higher concentration.

Assuming that all donors are ionized and all acceptor states are filled:

$$N_d - N_a = n_0 - p_0$$

To determine the main type of a compensated semiconductor evaluate:

$$k = |N_a - N_d|$$

If:

1. $n_0 = k$ then it is strongly n-type.
2. $p_0 = k$ then it is strongly p-type.

You can find the other concentration by:

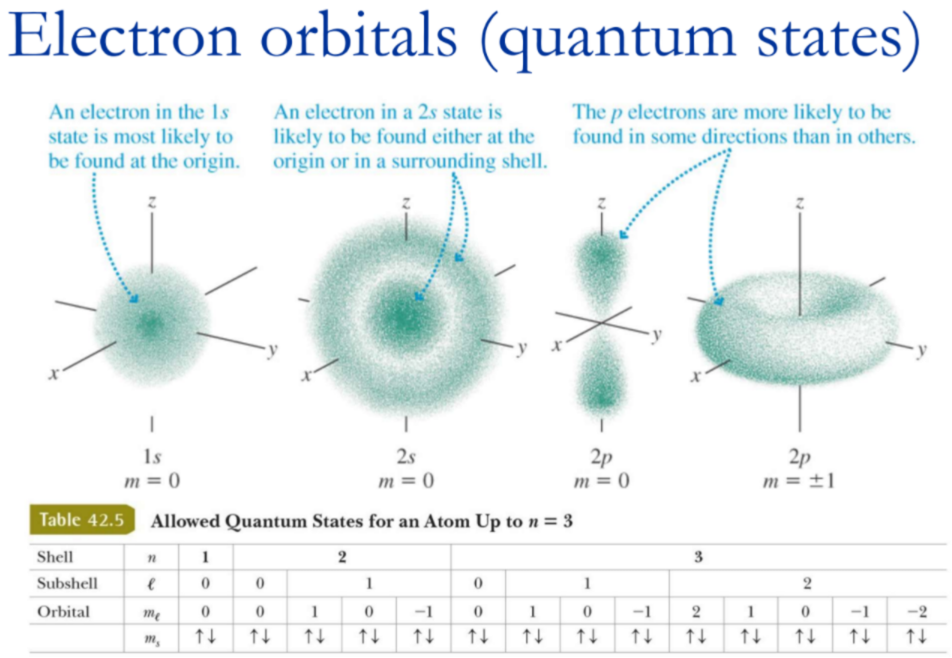
$$n_0 p_0 = n_i^2$$

Lesson 8: Band Structure

An electron in an atom is said to exist in a quantum state, defined by four variables:

1. Principle quantum number n.
2. Orbital angular momentum number ℓ.
3. z-component of orbital angular momentum number m.
4. Spin (either up or down).

Electrons in different quantum states have different orbital patterns:



Electron orbitals (quantum states)

An electron in the 1*s* state is most likely to be found at the origin.

An electron in a 2*s* state is likely to be found either at the origin or in a surrounding shell.

The *p* electrons are more likely to be found in some directions than in others.

Table 42.5 **Allowed Quantum States for an Atom Up to $n = 3$**

Shell	n	**1**	**2**				**3**								
Subshell	ℓ	0	0	1			0	1			2				
Orbital	m_ℓ	0	0	1	0	−1	0	1	0	−1	2	1	0	−1	−2
	m_s	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓	↑↓

These quantum states are notated:

$$n\ell^{\#}$$

Where:

1. $\ell = 0 \implies s$
2. $\ell = 1 \implies p$
3. $\ell = 2 \implies d$

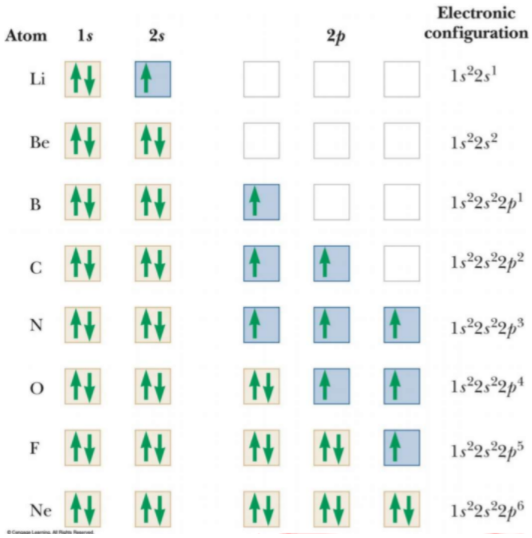
... and # is the number of electrons in the subshell:

1. s can hold 2 electrons.
2. p can hold 6 electrons.
3. d can gold 10 electrons.

When considering multi-electron atoms. Interactions between electrons affect the energy of the states. Electrons repel each other with some potential energy.

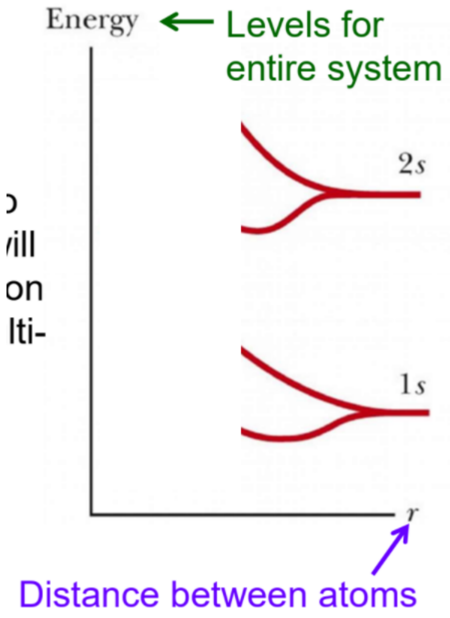
The Pauli Exclusion Principle states that two electrons cannot occupy the same quantum state simultaneously in one system. For a single atom, we use quantum numbers to describe the electrons, but for solids we use the energy band idea.

At $T = 0$ electrons will be at the lowest energy state available to them. When adding electrons to an atom, the energy levels will fill from the ground state up. States within a given subshell (s, p or d) have slightly different energy levels.

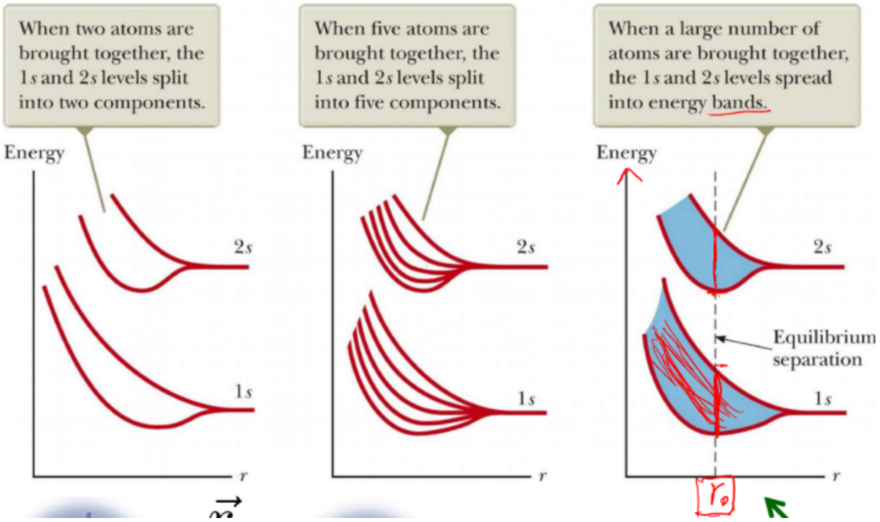


Atom	1s	2s	2p			Electronic configuration
Li	↑↓	↑				$1s^2 2s^1$
Be	↑↓	↑↓				$1s^2 2s^2$
B	↑↓	↑↓	↑			$1s^2 2s^2 2p^1$
C	↑↓	↑↓	↑	↑		$1s^2 2s^2 2p^2$
N	↑↓	↑↓	↑	↑	↑	$1s^2 2s^2 2p^3$
O	↑↓	↑↓	↑↓	↑	↑	$1s^2 2s^2 2p^4$
F	↑↓	↑↓	↑↓	↑↓	↑	$1s^2 2s^2 2p^5$
Ne	↑↓	↑↓	↑↓	↑↓	↑↓	$1s^2 2s^2 2p^6$

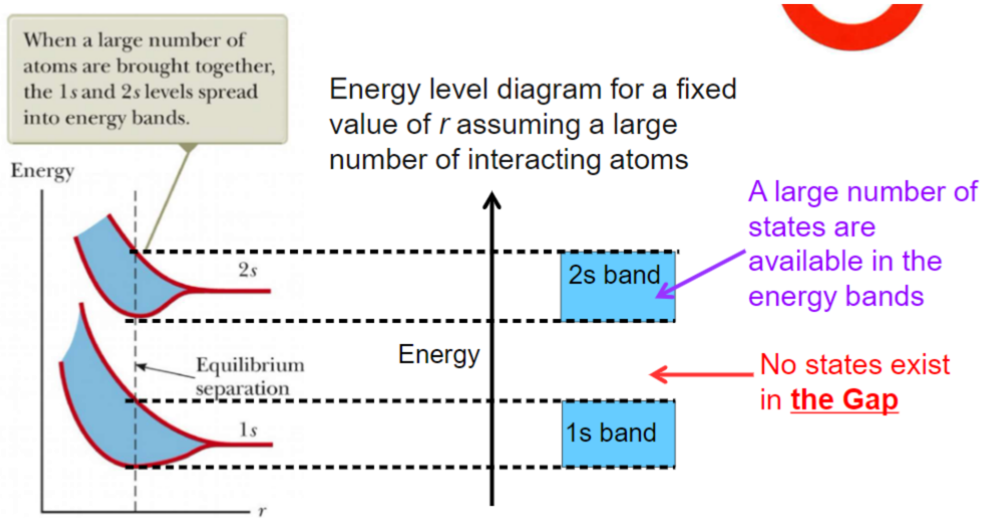
Considering the energy levels within a two atom system, we can see plot their distance from each other (r) and the energy of each atom in the system.



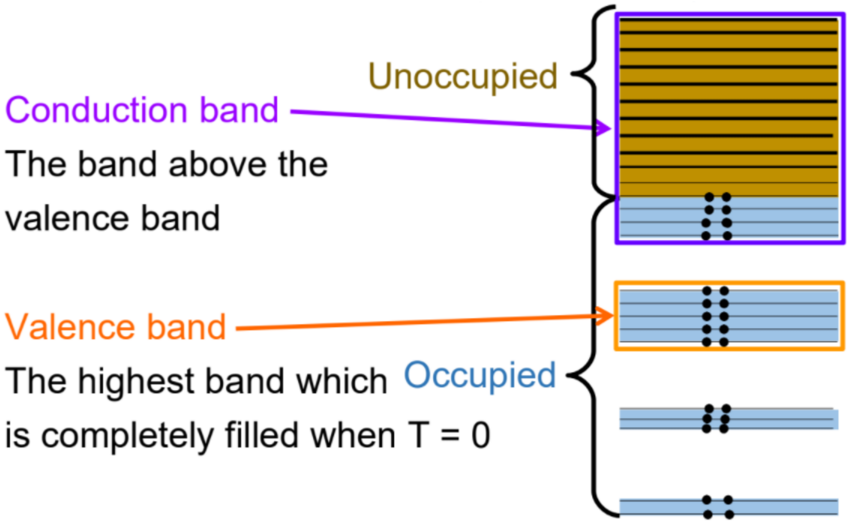
We can see that as you bring the atoms together, the energy level split into two parts for each atom. As you add more atoms, the energy levels split into more parts. Then when you consider a solid, there are so many breaks that you just consider it to be an energy band:



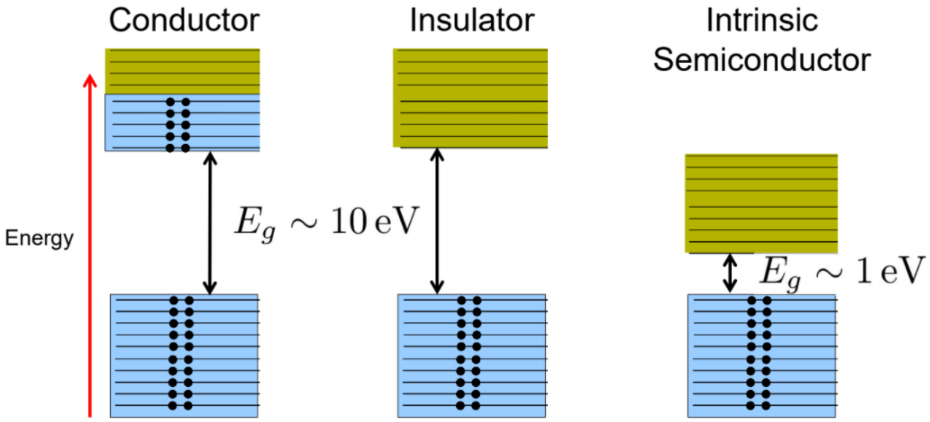
We then consider the energy levels at the equilibrium separation, which is how we get the energy band diagrams:



An energy band in a solid consists of a large number of energy levels so closely spaced that they may be regarded as a continuous band. We focus mainly on two specific bands in the band diagram:

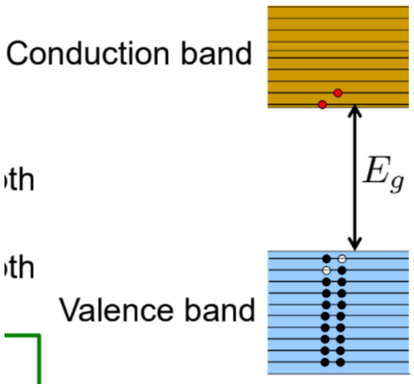


We can quantify the difference between insulators and conductors, as well as see how semiconductors work (all at $T = 0K$):



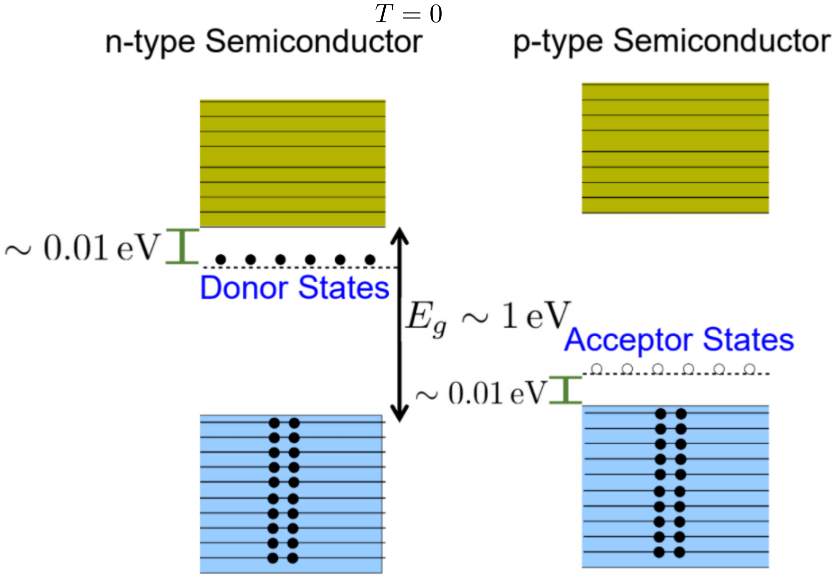
At $T = 0$ electrons have no kinetic energy. To create a current flow from the valence band to the conduction band you can either apply an electric field to exert a force on the electrons, or increase their kinetic energy (temperature). For a current to flow electrons must be excited and moved up to an empty state, which is easy in a conductor since the next available energy level is a very small distance away. This is hard to do in a insulator or a semiconductor as the electron would have to cross the gap to get to the next energy level.

An intrinsic semiconductor at nonzero temperature has free electrons in the conduction band and holes in the valence band:

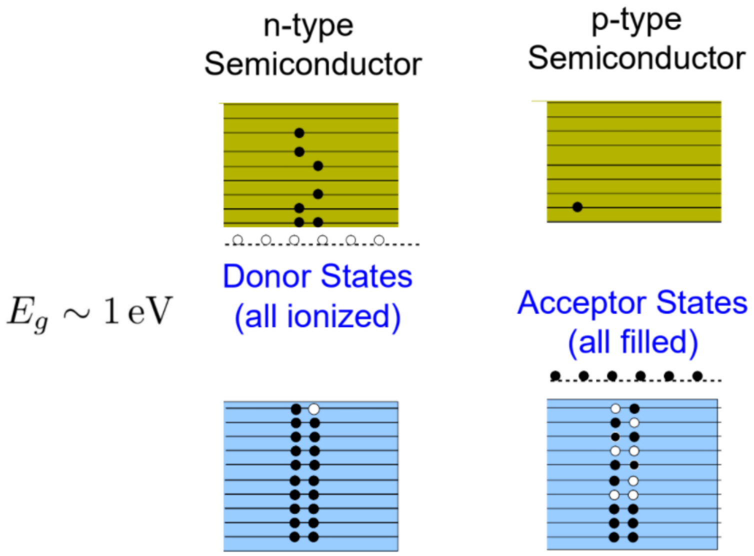


We can then consider doped semiconductors. The donor atoms in an $n-$type semiconductor exist in a stater just below the conduction band, and the acceptor states exist just above the valence band. Then at non-zero temperatures, all the donor states ionize, and all acceptor states are filled, moving their prior particles into the closest band:

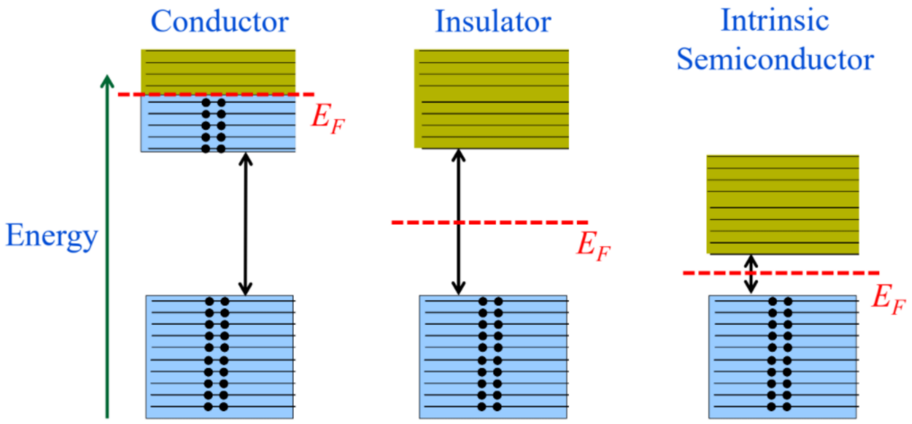
$T = 0$



$T > 0$



At $T = 0$ the Fermi energy (E_F) is the boundary between the filled and empty energy states:



Lesson 9: Fermi Dirac Statistic

We know what at $T = 0$ all the energy levels above the fermi energy are empty, and all the energy levels below are filled. But at non-zero temperatures we need to have a way to know how many energy states are filled. In general the following is true:

(number density of electrons with energy E) =

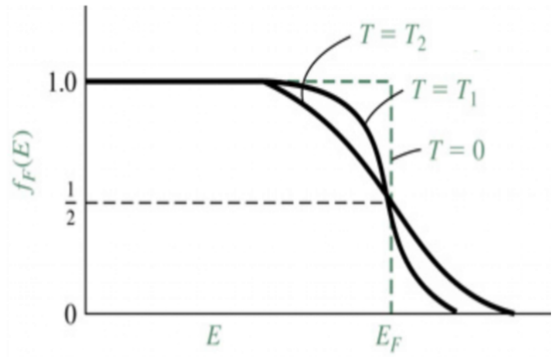
(number density of state at energy E)(probability of a state of energy E being occupied)

The final factor is defined as the *Fermi-Dirac function* for some energy E ($f_F(E)$) and is defined as:

$$f_F(E) = \frac{\text{number density of electrons with energy E}}{\text{number density of states at energy E}}$$

Mathematically it is defined as:

$$f_F(E) = \frac{1}{1 + e^{\frac{E - E_F}{k_B T}}}$$



Notably:

$$f_F(E_F) = \frac{1}{2}$$

Assuming there are states at all energy levels.

1. E_c is defined as the energy at the bottom of the conduction band.
2. E_v is defined as the energy at the top of the valence bond.

This means:

$$E_g = E_c - E_v$$

... and for an intrinsic semiconductor:

$$\frac{E_g}{2} = E_c - E_F$$

$$\frac{E_g}{2} = E_F - E_v$$

The probability of finding a hole at an energy level E is the probability of not finding an electron, as in:

$$1 - f_F(E) = \frac{1}{1 + e^{\frac{E_F - E}{k_B T}}}$$

Effective Mass

Electrons move through a material differently than they move in free space, they act as though they have different mass called the *Effective mass (density of states)*.

1. $m_n^* =$ effective mass of electrons
2. $m_p^* =$ effective mass of holes
3. $m_0 =$ electron mass in vacuum

These variables can be treated as temperature independent for this course.

We also define some new variables (found on tables):

1. N_c is the effective density of states in the conduction band.
2. N_V is the effective density of states in the valence band.

Carrier Concentration Formulas

By using the Fermi-Dirac function, and some integration, we can get some very important equations relating to semiconductors (doped or undoped):

$$\boxed{n_0 = N_c e^{-\frac{E_c - E_F}{k_B T}}}$$

$$\boxed{p_0 = N_V e^{-\frac{E_F - E_V}{k_B T}}}$$

$$\boxed{n_0 p_0 = N_c N_V e^{-\frac{E_g}{k_B T}}}$$

$$\boxed{n_i = p_i = \sqrt{N_c N_v} e^{-\frac{E_g}{2k_B T}}}$$

The above equations can be applied to any semiconductor (doped or not) in thermal equilibrium.

$$\boxed{n_0 = n_i e^{\frac{E_F - E_{Fi}}{k_B T}}}$$

$$\boxed{p_0 = n_i e^{\frac{-(E_F - E_{Fi})}{k_B T}}}$$

Effective Density of States Formulas

In conduction band:

$$N_C = \frac{2.5 \times 10^{19}}{cm^3} \left(\frac{m_n^*}{m_e}\right)^{3/2} \left(\frac{T}{300K}\right)^{3/2}$$

$$N_C(T) = N_C(300K) \cdot \left(\frac{T}{300K}\right)^{3/2}$$

In valence band:

$$N_V = \frac{2.5 \times 10^{19}}{cm^3} \left(\frac{m_p^*}{m_e}\right)^{3/2} \left(\frac{T}{300K}\right)^{3/2}$$

$$N_V(T) = N_V(300K) \cdot \left(\frac{T}{300K}\right)^{3/2}$$

Note that the effective density of states is temperature dependent.

Given that some material has $N_c = N_V$, this means the effective masses of electrons and holes in the material are the same.

Fermi Energy Position Formulas

The following formulas relate to the position of the Fermi energy level in a doped semiconductor:

$$E_C - E_F = k_B T \ln\left(\frac{N_C}{n_0}\right)$$

$$E_F - E_V = k_B T \ln\left(\frac{N_V}{p_0}\right)$$

$$E_{mid-gap} = \frac{1}{2}(E_C + E_v)$$

$$E_{Fi} = E_{mid-gap} + \frac{3k_B T}{4} \ln\left(\frac{m_p^*}{m_n^*}\right)$$

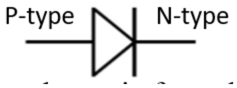
$$E_F = E_{Fi} + \frac{k_B T}{2} \ln\left(\frac{n_0}{p_0}\right)$$

In general, for an $n-$type semiconductor $E_F > E_{Fi}$, and for a $p-$type semiconductor $E_F < E_{Fi}$

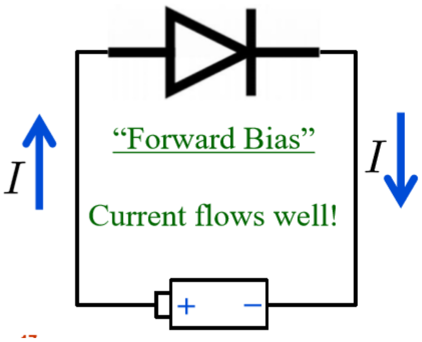
Lesson 12: Unbiased PN Junction

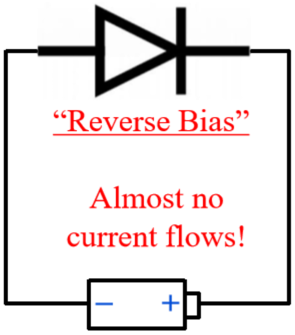
The PN junction is the simplest application of all the math and physics we just did. The PN junction is used in many electronic devices.

A PN junction is formed by joining a p-type and an n-type semiconductor (with potentially different doping). We also call this device a *diode* for the time being, the circuit element symbol is shown on the right:



The diode only allows current to travel from p-type to n-type, in the other direction very little current travels.

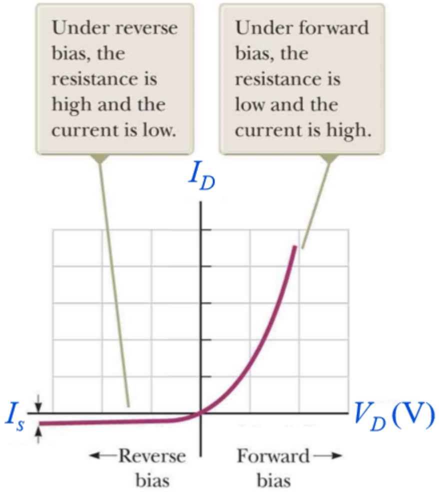




The Diode Equation

The current through the diode (I_D) can be described as a function of the voltage across it V_D.

$$I_D = I_S\left(e^{\frac{V_D}{V_t}} - 1\right)$$



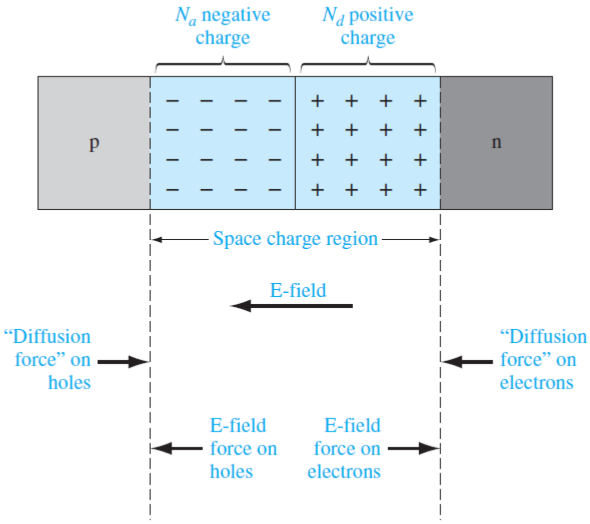
Where:

1. $$V_t = \frac{k_B T}{e}$$
2. I_s is the maximum current through the diode when it is reverse biased.

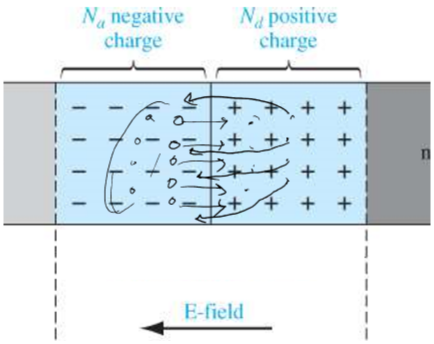
If $V_D > 0$ the diode is forward biased, if $V_D < 0$ the diode is reverse biased. positive I_D is defined to go from $p-$type to $n-$type.

When you make a *pn*-junction, diffusion occurs across the barrier. Electrons move from the n-type material, and holes move from the p-type material to the other to annihilate with each other. In this process, photons are emitted when electrons move from the conduction band to the valence band to annihilate with a hole.

The region around the separator between the materials becomes a *depletion region*, which is where the n-type material has lost many electrons (exposing the positive donor atoms), and the p-type material has lost many holes (exposing the negative acceptor atoms).



This separation of charge causes an electric field from the n-type to the p-type, which equalises with the "diffusion force". We are interested in how long each side of the depletion region is:



Since the charge of the depletion region must be zero we get the equation:

$$N_a x_p = N_d x_n$$

To calculate the electric field within the depletion region, we treat each block of the depletion region as infinite plates of charge, meaning:

$$\left|\vec{E}\right| = \frac{Q_{tot}}{2A\kappa\varepsilon_0}$$

Remember since we are in a material we need to use the dielectric constant.

$$\left|\vec{E}\right| = \frac{-eN_a}{\kappa\varepsilon_0}(x + x_p) \text{ for } -x_p \leq x \leq 0$$

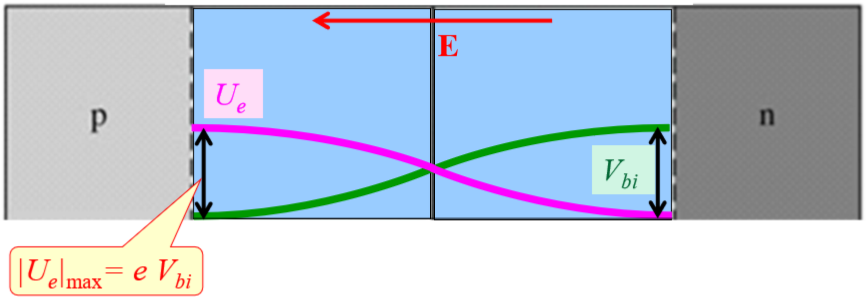
$$\left|\vec{E}\right| = \frac{-eN_d}{\kappa\varepsilon_0}(x + x_p) \text{ for } 0 \leq x \leq x_n$$

$$\left|\vec{E}\right|_{max} = \frac{-eN_a x_p}{\kappa\varepsilon_0}$$

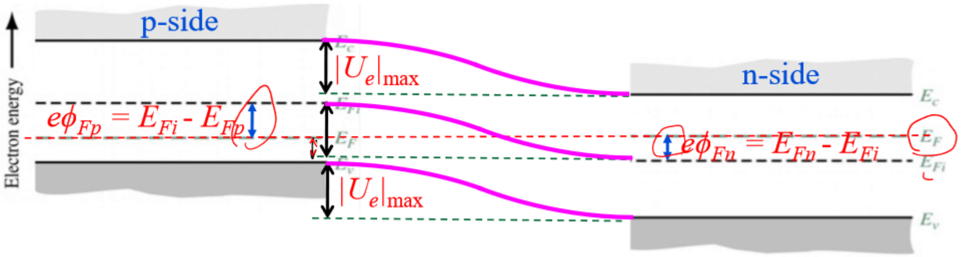
We then define V_{bi} to be the built in potential of the pn-junction, and W to be the width of the depletion region ($W = x_n + x_p$), giving us:

$$V_{bi} = \frac{1}{2}\left|\vec{E}_{max}\right| W1$$

We can then see the different potentials for an electron (purple) and a hole (green):



Before forming a junction, the p-type material has a lower E_F than the n-type. After forming the junction the depletion region forms and the Fermi energy levels align when in equilibrium. The energy bands shift vertically to enforce these equilibrium conditions:



(Here $\phi_{Fp} = \Delta V$)... as you can see:

$$|U_e|_{max} = eV_{bi}$$

$$eV_{bi} = E_{Fn} - E_{Fp}$$

We can then derive another formula for V_{bi}:

$$V_{bi} = \frac{k_B T}{e} \ln\left(\frac{N_d N_a}{n_i^2}\right)$$

Where:

1. N_d is the net donor concentration on the n-doped side:

$$N_d = N_{dn} - N_{an}$$

2. N_a is the net donor concentration on the p-doped side:

$$N_a = N_{ap} - N_{dp}$$

We finish this section with a few more formulas:

$$x_p = \left(\frac{2\kappa\varepsilon_0 V_{bi}}{e(N_a + N_d)} \frac{N_d}{N_a}\right)^{\frac{1}{2}}$$

$$x_n = \left(\frac{2\kappa\varepsilon_0 V_{bi}}{e(N_a + N_d)} \frac{N_a}{N_d}\right)^{\frac{1}{2}}$$

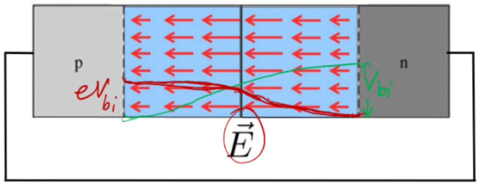
$$W = x_n + x_p = \left(\frac{2\kappa\varepsilon_0 V_{bi}(N_a + N_d)}{eN_a N_d}\right)^{\frac{1}{2}}$$

$$\left|\vec{E}\right|_{max} = \frac{2V_{bi}}{W}$$

Summary of the Diode Biases

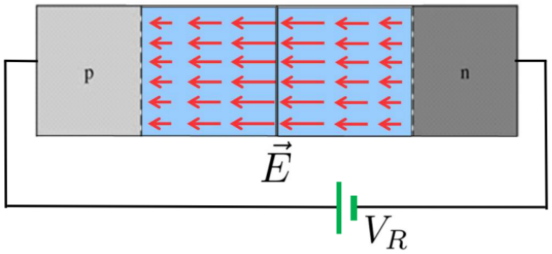
Unbiased (in a zero bias)

Unbiased diodes occur when the two sides of the junction are connected by a short-circuit. There is ideally a zero potential difference between the two sides of the pn junction because of the wire.



Forward Biased

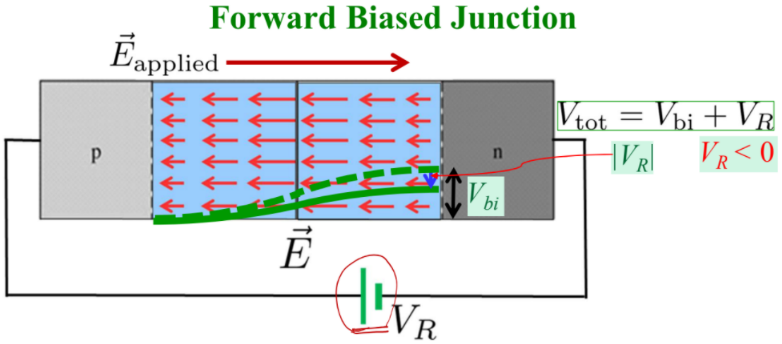
The p side of the pn junction is connected to the positive side side of the voltage source. The voltage source has a positive potential difference of V_R.



In forward bias, the potential difference within the pn junction decreases, which is what allows for the current to flow.

$$V_{tot} = V_{bi} + V_R$$

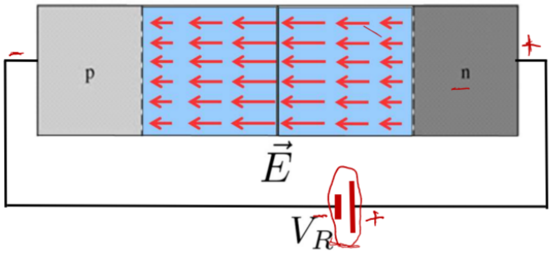
... V_{tot} is the voltage that the current carriers must overcome in order for the current to flow. From the perspective of the junction, $V_R < 0$.



In forward bias, the current is large due to the fact that the majority current carriers are flowing.

Reverse Biased

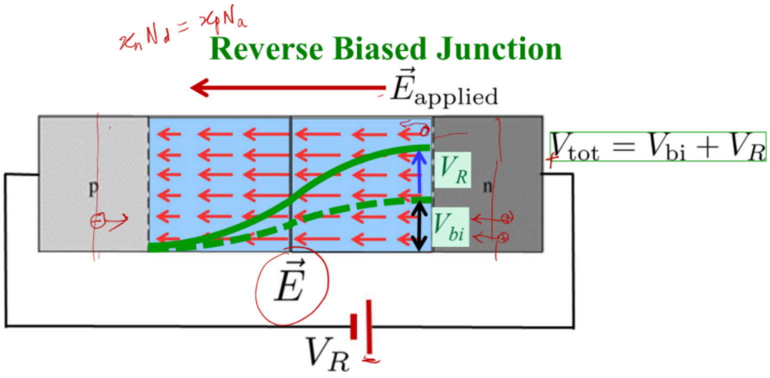
The n side of the pn junction is connected to the positive side of the voltage source. The voltage source has a positive potential difference of V_R.



In reverse bias, the potential difference within the pn junction increases, which is what stops the current to flow.

$$V_{tot} = V_{bi} + V_R$$

... V_{tot} is the voltage that the current carriers must overcome in order for the current to flow. From the perspective of the junction, $V_R > 0$.



In the reversed bias, only the minority current carriers can flow, which is why the reverse current is so low.

When working with a biased pn-junction, you can replace V_{bi} in certain formulas with $V_{bi} + V_R$. Recall that:

- Forward biased:
$$V_R < 0$$
- Reversed biased:
$$V_R > 0$$

Capacitors

Capacitors are devices which are made of two conducting plates separated by a dielectric (insulating) material. The conducting plates become charged, each plate carries some charged particles which created a potential difference. Experimentally:

$$Q \propto \Delta V$$

The proportionality constant depends on shape and separation of the conductors when in a vacuum.

We call this proportionality constant C for capacitance, which represents the capacitors ability to store charge.

$$C \equiv \frac{Q}{\Delta V}$$

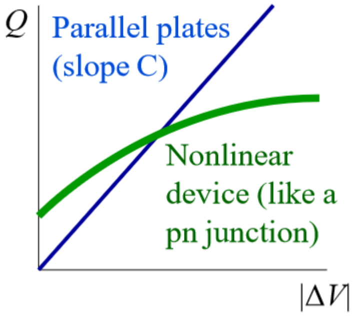
The capacitance depends on the dielectric and spacing of the plates:

$$C = \frac{\kappa \varepsilon_0 A}{d}$$

κ is the dielectric constant of the material. We will next define a new type of capacitance, so remember that this formula is just for *parallel plate capacitors*.

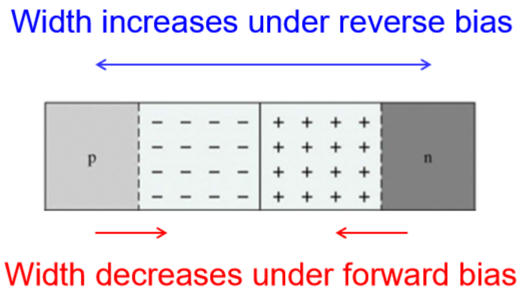
Capacitance is a property of the material, and does not change with changes in voltage.

Many devices have their charge Q being non-linearly proportional to ΔV. In this case, we cannot say the capacitance is the ratio, rather it is the derivative:



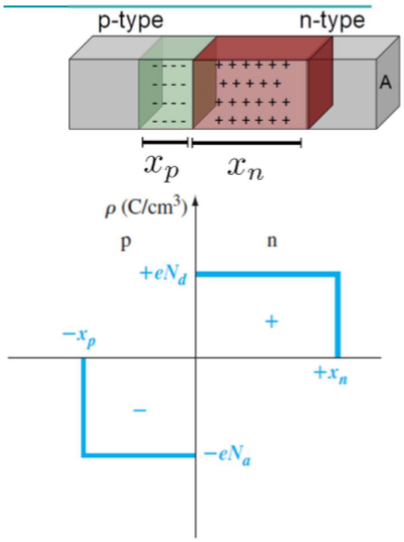
$$C = \frac{dQ}{d|\Delta V|}$$

A pn junction acts like a parallel plate capacitor where the distance between the plates varies as the voltage changes.



Charge Diagrams

The following is an example of a charge diagram:



The area of each square is the charge density per area of the depletion region. Multiply this by the area of the junction and you get the total charge in each region, and since they must be equal (net neutral charge), we get:

$$eN_dx_nA = eN_ax_pA$$

We can then derive a formula for the capacitance of a pn junction:

$$\boxed{C = A\left(\frac{e\kappa\varepsilon_0 N_a N_d}{2(V_{bi}+V_R)(N_a+N_d)}\right)^{\frac{1}{2}}} = \frac{\kappa\varepsilon_0 A}{W}$$

One-Sided Junction

This is a special case where:

- p^+n junction: $N_a >> N_d$.
- n^+p junction: $N_d >> N_a$.

In either case:

$$W \approx x_{n/p}$$

For a p^+n junction:

$$C = A\left(\frac{e\kappa\varepsilon_0 N_d}{2(V_{bi}+V_R)}\right)^{\frac{1}{2}}$$

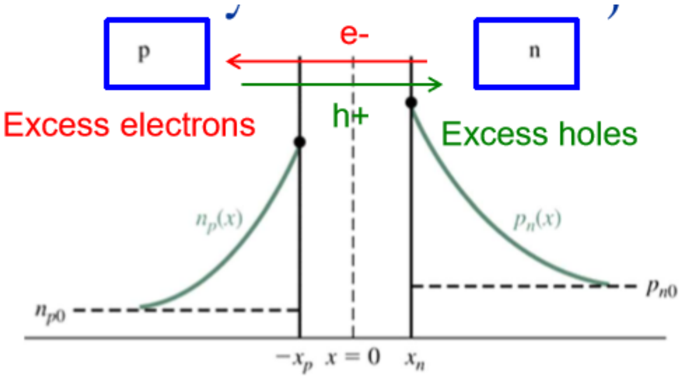
For a n^+p junction:

$$C = A\left(\frac{e\kappa\varepsilon_0 N_a}{2(V_{bi}+V_R)}\right)^{\frac{1}{2}}$$

Minority Carrier Injection

In thermal equilibrium, the rate of generation of electron-hole pairs is equal to the rate of recombination. The length of time it takes for a hole to recombine with an electron, and vice versa is called the carrier life time denoted τ_p, τ_n.

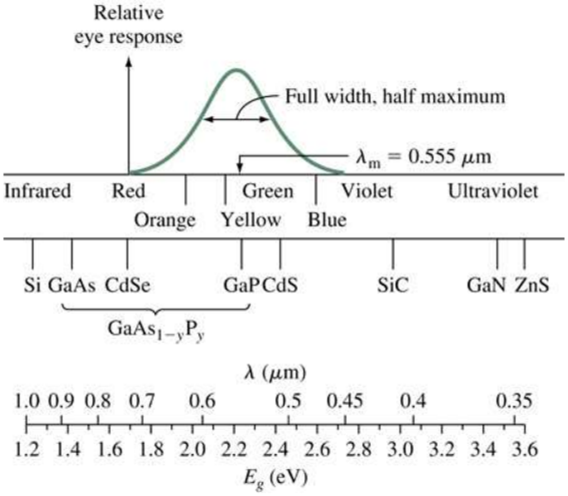
In a pn junction, majority carriers flow across the depletion region, but once they've entered the other semiconductor, they are minority carriers (which have been "injected"). Recombination begins, but more charges always come and so there are more minority carriers on the barrier of the semiconductor and depletion region on both sides.



As you go further into each semiconductor, there are less and less minority carriers until you reach some equilibrium concentration.

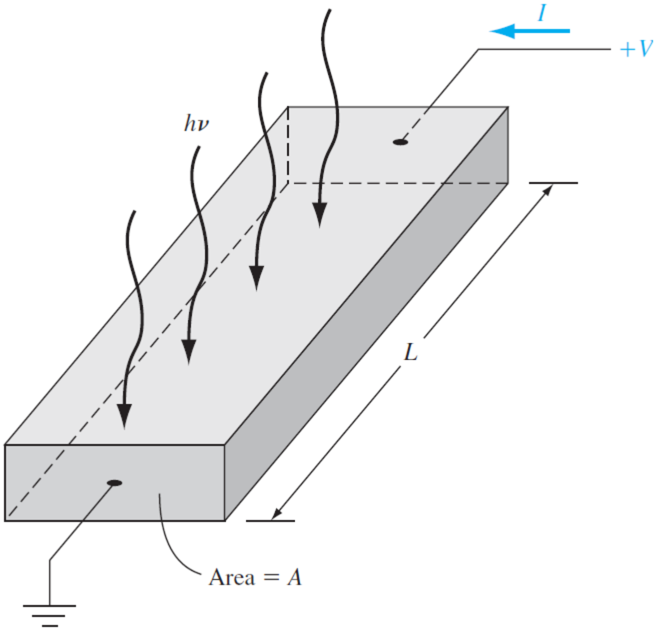
Lesson 14: Photoelectronic Devices

When a photon hits a semiconductor, electrons are brought up to a higher energy state. The energy of the photon must be at least the band gap energy. The electron will then go to a lower energy level and release a photon which depends on the band gap energy of the semiconductor:



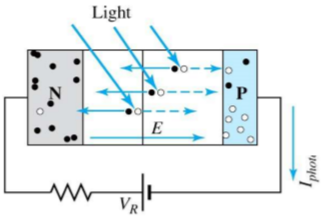
Photoconductor

A material becomes more electrically conductive due to the absorption of photons. A photoconductor is a semiconductor made of a single type, which reacts to photons. Increased free charge carriers due to the photons means increased conductivity. With a constant voltage applied the current increases.

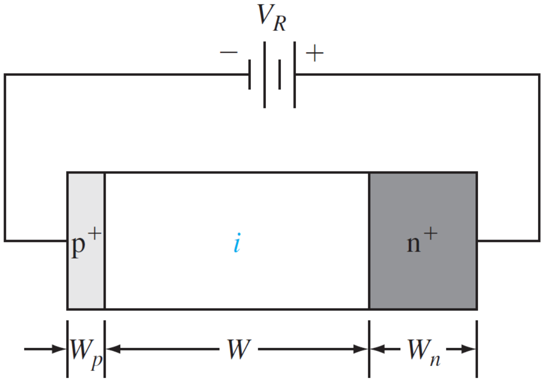


Photodetector

When light hits the depletion region of a pn-junction under reverse bias, the electron will move to the $n-$type region and the hole will move to the $p-$type region. This induces a current in the pn-junction called the *photo current*. The reverse bias current is very small compared to the photocurrent, and so it can be easily detected.



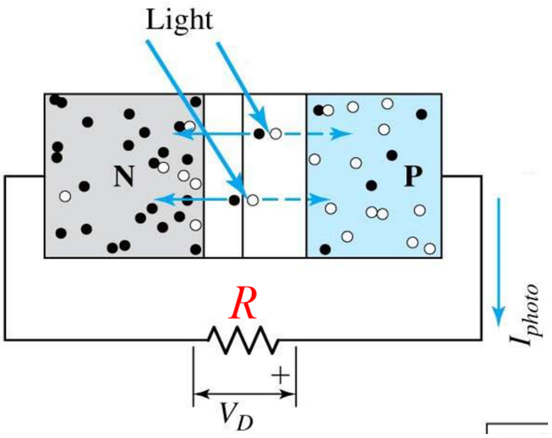
You want the width of the depletion region to be as wide as possible to allow the most photons to hit the junction, for this reason a *PIN* junction is used by placing an intrinsic semiconductor between the two types of semiconductors, as in:



Solar Cell

The solar cell is the only optoelectronic device we study quantitatively.

The solar cell is the same as a photodetector except without a source:



The photocurrent (I_L) in the reverse bias direction causes a voltage drop across the resistor. This voltage drop induces a forward bias current (I_D) which goes opposite the photocurrent. The net current is therefore:

$$I_{tot} = I_L - I_D$$

The net current is always in the reverse bias direction. Remember we have a formula for I_D, so the current becomes:

$$I_{tot} = I_L - I_S\left(e^{\frac{eV_D}{k_BT}} - 1\right)$$

The voltage difference V_D is equal to the voltage across the resistor:

$$V_D = RI_{tot}$$

The open circuit voltage of the solar cell, which occurs when $R \to \infty$ is:

$$V_{OC} = \frac{k_BT}{e}\ln\left(1 + \frac{I_L}{I_S}\right)$$

Heterojunction Solar Cell

This type of solar cell is created by combining semiconductors with different bandgap energies. This improves sensitivity to photon energies.

Light-Emitting Diodes

Light is emitted from a pn-junction when electrons recombine with holes at the electorneutral region in forward bias. The energy of the light is the gap energy of the semiconductor. All diodes release light but LEDs are specially designed to do so efficiently and in the visible spectrum.

Conclusion

My notes for this course will remain incomplete. To see notes on the final content in this course, I recommend visiting the following link to see notes compiled by a fellow student:

Click on this text to access the completed set of notes.

I hope these notes were helpful, good luck on the exam!
- Adam Szava